Rotor Hovering Performance Using the Method of Fast Free Wake Analysis

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Experimental investigations have shown that the bound circulation distribution on a rotor blade is critically dependent on the location of the vortices in the wake. Consequently, in order to optimize rotor performance, compute blade loads, and determine acoustic signatures, it is necessary to use analytical methods which are capable of predicting wake geometry. The factors which contribute to the complexity of the problem both in hovering and forward flight are discussed. A simplified approach to a free wake analysis of rotors in hover is outlined and the analytical results compared with experimental data for two- and four-bladed rotors.

Introduction

HELICOPTER rotor aerodynamic analysis remains one of the more challenging of the unsolved problems of modern fluid mechanics. Although solutions are not required for successful design, as evidenced by the growing use of the helicopter, the availability of a consistent aerodynamic theory for the rotor could lead to formal rather than heuristic design optimization and would help to reduce the costly flight testing required to minimize vibration, noise and rotor fatigue loads. This paper will discuss some of the unsolved problems of rotor aerodynamics and suggest a simplified approach to defining the complex vortex structure of the rotor and its wake. Such simplification could facilitate the expansion of the mathematical models to include, for example, certain important real fluids effects.

Geometric Complexity

In hovering flight the lifting rotor blade generates a spiralling, rapidly contracting wake which, unlike a lifting wing where the wake is transported rapidly downstream, remains in the vicinity of the rotor at least for the initial spiral. The bound circulation distribution along a following blade is strongly influenced by this wake because of its proximity to the blade and rapid initial contraction. Figure 1 sketches the geometry at the crucial first encounter of a blade with the rolled up tip vortex in the wake. Figure 2 shows a typical bound circulation distribution and wake geometry in hovering flight. The wake continues to distort under the influence of the self-induced velocities as it descends, which complicates the computation of induced velocities both at the blade and in the wake.

In the forward flight case the wake geometry may be somewhat simplified since only the first spiral has any appreciable influence on the blade loads, further spirals being carried away from the rotor as it advances. On the other hand, the forward flight case is complicated by the existence of time varying airloads requiring the inclusion of a shed, as well as a trailing wake structure and the use of nonstationary flow theory. In the case of hovering flight, because of symmetry, the flow is essentially stationary when viewed in the rotating system.

Several alternative approaches to handling the geometric complexities outlined above have been proposed. The simplest is to assume a "rigid" wake in which the wake displacements are determined by the velocities existing at the points on the blade from which the wake originally trailed. This procedure is marginally adequate for predicting airload distributions in forward flight, but results in appreciable error in circulation distribution in hovering flight. An improvement is the "prescribed wake" technique for hovering flight in which the wake geometry is determined experimentally and then used to predict airloads. The consistency in the experimentally determined wake geometries found by different investigators lends credence to the results obtained using this method. Its application, however, is limited to rotors having geometries similar to the test articles and the method can not therefore be used for global optimization of rotors with arbitrary geometries. Finally, "free wake" techniques have been developed in which the rotor wake is allowed to assume any position as determined by the time history of induced velocities everywhere in the wake. Their complexity, however, discourages the inclusion of those real fluid effects which experimental evidence indicates may be of prime importance.

It is one of the purposes of this paper to show that these geometric complexities may be appreciably reduced by using simplified models leading to a fast free wake analytical technique, thereby paving the way for the inclusion of more complex flow representations.

Analytical Complexity

No closed-form solution exists, even for an idealized rotor, in forward flight. An empirical solution based on momentum theory for an actuator disk which satisfies the boundary conditions in hover and at high forward speed (when the rotor may be approximated by a circular wing) is in current use for performance estimation. In hovering flight the Rankine-Froude momentum theory first proposed in the 1860s is still the standard method for performance analysis. The Prandtl-Glauert vortex theory developed for propellers, in which the vortex spiral is represented by a semi-infinite cylinder, is sometimes used, but this theory is not applicable to the case of a hovering rotor since its basic assumption of induced velocities small compared to the forward velocity is obviously not satisfied for the hovering case where the induced flow is the only axial flow through the rotor. Consequently, certain simplifying assumptions, such as the neglect of wake contraction in computing the velocities induced by the vortex cylinder, are inadmissible.

Although the momentum theories for hovering and forward flight have proven to be simple and satisfactory methods for performance estimation, they are based on an assumed infinite number of blades and can not therefore be used for predicting blade loads, these loads being critically dependent

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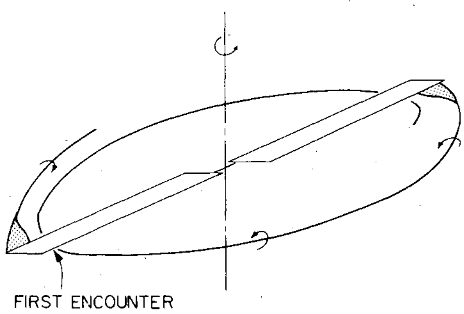

Fig. 1 First blade vortex encounter.

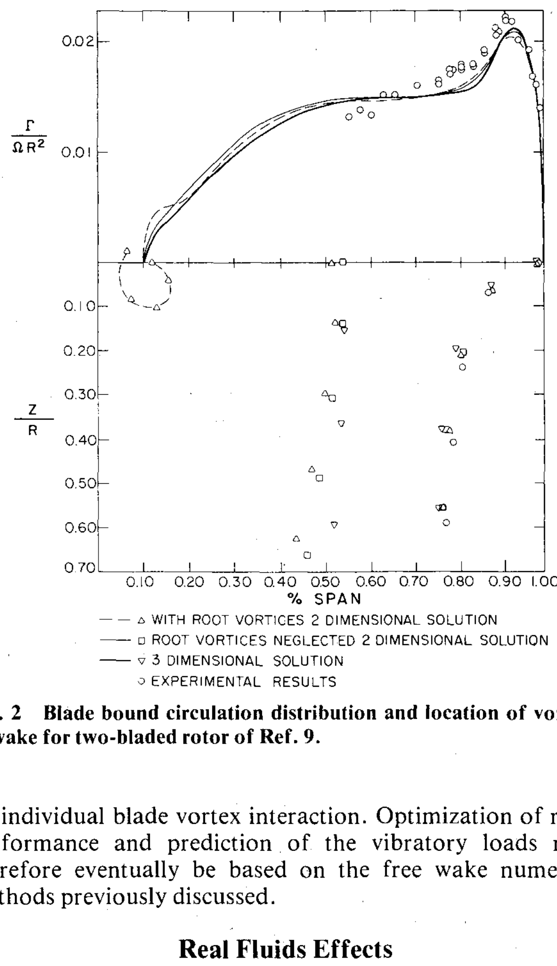

Fig. 2 Blade bound circulation distribution and location of vortices in wake for two-bladed rotor of Ref. 9.

on individual blade vortex interaction. Optimization of rotor performance and prediction of the vibratory loads must therefore eventually be based on the free wake numerical methods previously discussed.

Real Fluids Effects

The importance of real fluids effects became apparent when attempts were made to predict blade airloads in forward flight using a complete wake representation. Initial results obtained using a rigid wake for comparison with the experimental airloads of Ref. 1 (one of the first comprehensive flight test programs in which blade pressure distributions were measured) showed reasonable agreement at the blade tip but deteriorated as the radius decreased. For example, Fig. 3, taken from Ref. 1, shows a rapid variation of load in the computed results at the 85% span location near the 90 deg azimuth which did not appear in the test data. It was suspected that this and other discrepancies were due to the rigid wake assumptions. Consequently, the free wake analysis technique of Ref. 3 was developed. At the same time the wake geometry was determined experimentally with smoke tests on a wind tunnel model. Figure 4, taken from Ref. 3, shows that at least qualitatively the theory correctly predicted the observed geometry. However, when this theory was used to predict airloads, agreement was much worse than in the case of the rigid wake, as shown in Fig. 5. Vortex bursting had occasionally been noticed during the wind tunnel tests and

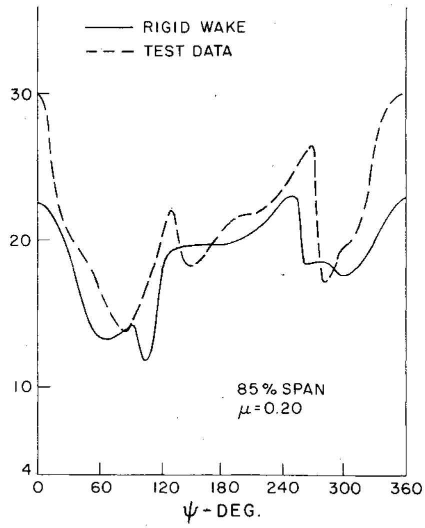

Fig. 3 Comparison of computed and measured flight airloads for four-bladed rotor of Ref. 1 in forward flight; rigid wake assumption.

reported by other investigators; therefore a rapid increase in core size after first encounter was added to the analysis. The lifting line representation used for the blade was also changed to a lifting surface representation at close blade vortex encounters using the theory of Ref. 4. Some improvement was realized, but it was only when vortex breakdown well ahead of first encounter was assumed did reasonable agreement result, as shown in Fig. 6. More recently, laser velocimeter tests have indicated that the premature bursting postulated in Ref. 3 does indeed occur, as shown in Fig. 7 from Ref. 5.

Another phenomenon involving separation at the leading or trailing edges of the blade appears to occur at close blade vortex encounters, possibly due to the resulting rapid spanwise variations in flow along the blade. Reference 6 demonstrated the occurrence of such premature separations experimentally and showed that the maximum incremental lift coefficient which could be generated by a close blade/vortex encounter was limited to from about 0.2 to 0.3. Figure 8, taken from Ref. 7, illustrates this phenomenon.

Tip losses are expected, particularly in the case of blades with rapid changes in geometry towards the tip. Rotor blades, unlike fixed wings, carry a high level of circulation near the tip (Fig. 2). The bound circulation therefore must leave the blade in a strong tip vortex. Part of the tip losses must be attributed to the as yet not fully understood flow characteristics at the tip as this strong tip vortex is formed. In addition, separation most likely occurs as the flow proceeds around the tip from the lower to the upper surface.

Finally, the vortices in the wake must contain viscous cores whose size may be estimated by the methods discussed in Ref. 8. In addition, the roll-up of the strong tip vortex results in distributed vorticity outside the core, as predicted by the Betz criteria of conservation of linear and angular momenta. These effects are shown in Ref. 9 to be small for a two-bladed rotor but may be appreciable for the four-bladed rotor discussed below.

Fast Free Wake Analysis

In order to explore these problems more fully it is desirable to identify simplifications which would reduce the computational effort required to determine the wake geometries. One such simplification was used for the forward flight case in Ref. 2 where the wake at encounter with a blade was replaced by an infinite vortex line. It was shown that this approximation gave excellent agreement with the more complete solutions in which the entire wake was modeled, while reducing computational time to about 2% of the

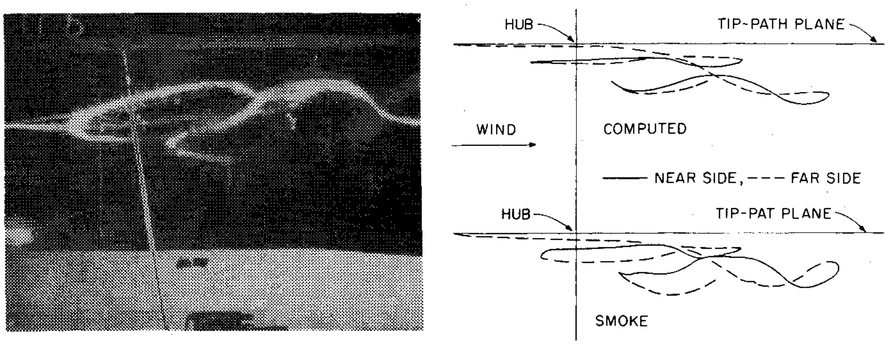

Fig. 4 Predicted free wake geometry and comparison with wind tunnel test, $\mu = 0.10$.

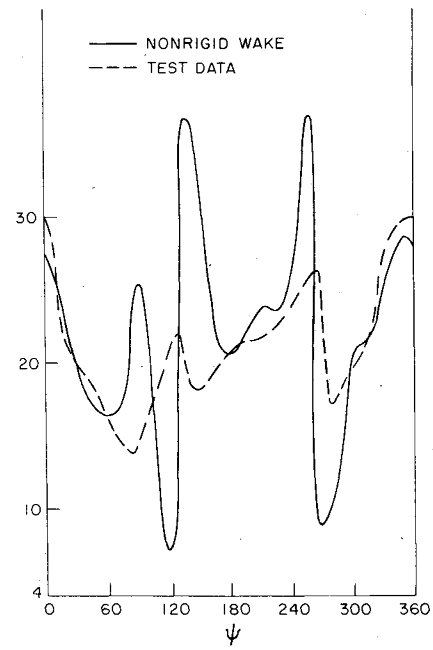

Fig. 5 Comparison of computed and measured flight airloads; free wake analysis.

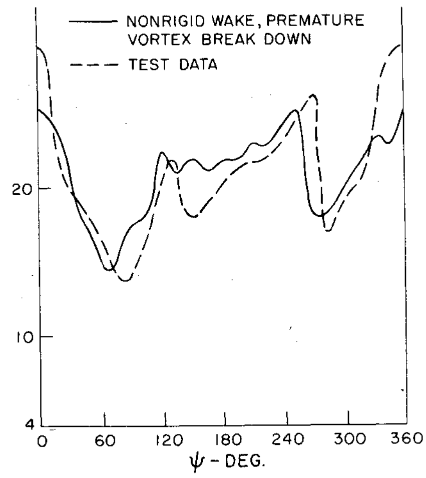

Fig. 6 Blade airloads computed with vortex breakdown assumption.

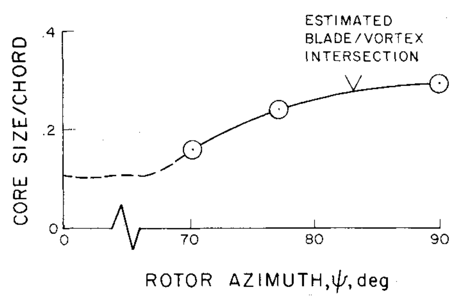

Fig. 7 Vortex bursting prior to blade/vortex intersection.[5]

original. This method is now being extended to include a free wake in forward flight.

In the meantime, the same approach was used in Ref. 10 to treat the more complex case of hovering flight. It was found that very simple models are also adequate for this case, for example, a quasi-two-dimensional one. The rotor wake is replaced by 1) a near wake consisting of semi-infinite vortex filaments attached to the blade in the plane of the rotor with the blade represented by a lifting line, 2) pairs of infinite line vortices below the rotor forming an intermediate wake, and 3) a far wake consisting of a pair of vortex sheets. This geometry is possible because of the symmetry of the rotor in hovering flight. Viewed from the side, this wake model appears as a two-dimensional model (Fig. 9). Wake displacements between any two vortices are determined from the vorticity transport law (vortex moves with the fluid) using the average of the velocities at the two vortices acting over a time increment corresponding to the individual blade passage. A similar model, but using vortex rings for the intermediate wake and vortex cylinders for the far wake, is shown in Fig. 10. For simplicity, only one of the series of rings and one cylinder formed by the tip vortex is shown, although others will exist inboard depending on the roll-up schedule assumed. The two-dimensional model results from replacing the rings by pairs of infinite line vortices, one of which is located below the blade in question. The model is thus applicable to a rotor with any number of blades.

Figure 2 taken from Ref. 9, shows results obtained using these models compared to the experimental data of Ref. 11 and the estimated wake positions of Ref. 12. It was assumed that the near wake rolled up into vortex filaments, conserving linear momentum in the process, according to various schedules determined by the bound circulation distribution. In this case a tip, root, and center vortex were postulated. It should be noted that although there is clear experimental evidence for the existence of a strong tip vortex, there is as yet none for the inner wake vortices. Until more experimental data becomes available, any assumed roll-up schedule for the center portion of the blade should be considered more as a convenient computational technique to account for the effects of the inboard trailing vorticity rather than as an exact prediction of the wake structure.

As expected, the center vortex descends at a more rapid rate than the tip vortex. Of particular interest is the root vortex, whose position is strongly influenced by the upflow at the center of the rotor frequently observed on hovering helicopters. The migration of this root vortex (with its attendant far wake) through the blade, causes computational difficulties resulting in slow and uncertain convergence. And yet its influence upon either the blade loading or rotor performance is negligible, as shown in Fig. 2. Furthermore, it is

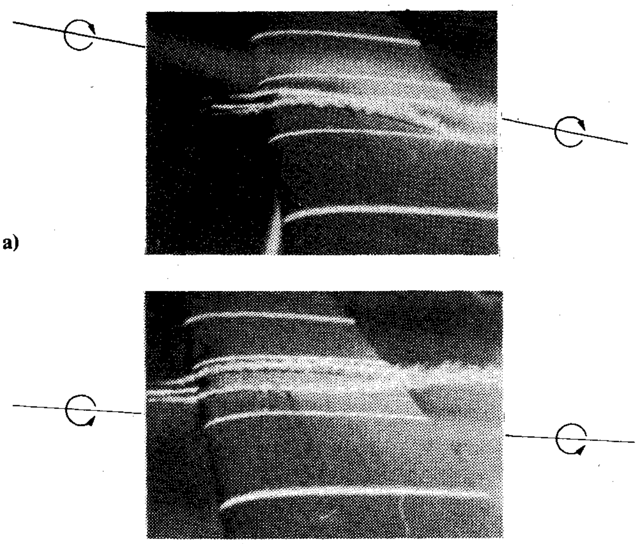

Fig. 8 a) Typical leading and b) trailing edge separation occurring as a result of close blade vortex interaction.

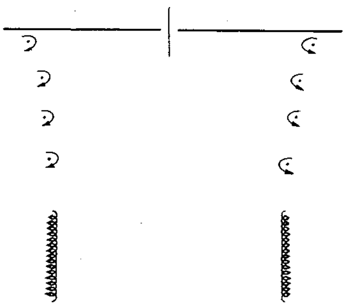

Fig. 9 Geometry of model using line vortices and vortex sheets to represent wake.

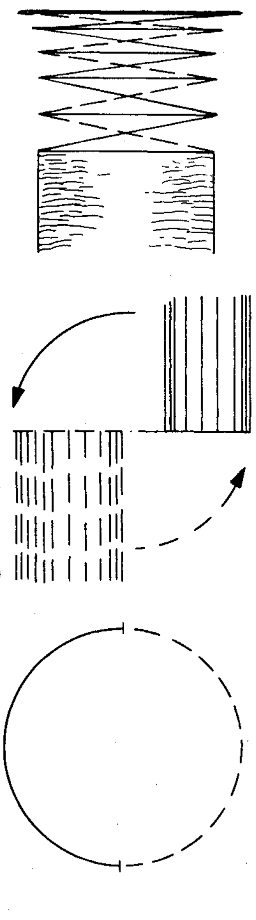

Fig. 10 Geometry of model using vortex rings and cylinders. a) Side view of rotor wake model showing intermediate and far wakes formed from vortex spiral—two blades, tip vortex only; ——blade 1; – – – blade 2. b) Plan view showing near wake. c) Formation of intermediate wake.

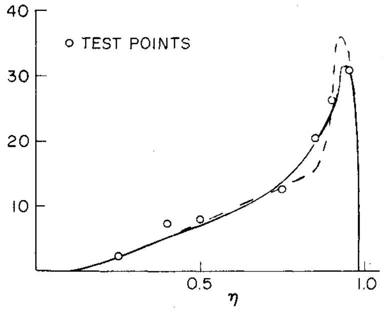

Fig. 11 Comparison of computed and measured airloads for four-bladed rotor of Ref. 1; ——free wake lifting surface solution with vortex breakdown; --- without vortex breakdown.

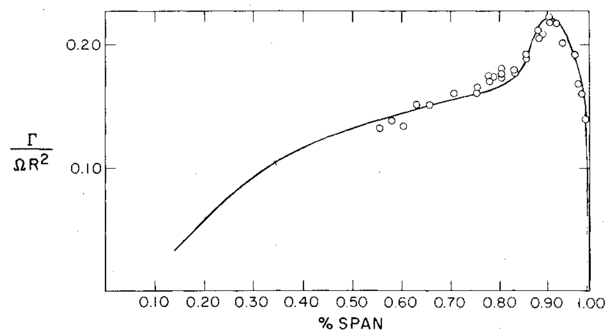

Fig. 12 Comparison of computed and measured airloads for two-bladed rotor of Ref. 9 with inner wake roll-up schedule as used for Fig. 11; $C_T = 0.00463$.

probable that this vortex does not exist in practice in the form postulated, since it trails into a region where the induced velocities, and hence the flow, are highly disturbed by the hub and root fittings. The resultant mixing and diffusion of the root vortices from all blades make their true contributions to the wake velocities uncertain. Since the effects of the root vortex on the solutions are in any case negligible, its strength has been set equal to zero for the other solutions discussed here.

Evidently the bound circulation on the blade is heavily influenced by wake contraction. Neither the inflow, nor the circulation distributions are close to the uniform values assumed in classical vortex theories for the "ideally twisted" rotor blade. The theoretical approach suggested here should be sufficiently simple computationally to permit heuristic and possibly formal techniques to be used for rotor performance optimization. Reference 9 contains a more detailed discussion of the theoretical development as well as the program description and listing for both models. Additional results are also given for a blade with a very highly tapered tip section (the ogee blade).

When this method is extended to four-bladed rotors, problems similar to those encountered in forward flight as discussed above occur. Unfortunately very little information is available on the airload distributions in hovering flight for four-bladed rotors and none with simultaneous measurement of wake geometries similar to the two-bladed results of Ref. 11. Figure 11 shows a comparison between the analytical and flight test results for the four-bladed rotor of Ref. 1. Best agreement was obtained when vortex bursting was assumed. The close proximity of the vortex to the blade at first encounter suggests that such breakdown may well occur. In order to allow for vortex bursting, an approximation suggested in Ref. 3 was used where the velocity induced by the vortex is reduced by a factor $1/[1+(\rho/a)^2]$, where a is the distance of the vortex from the blade and ρ is a factor representing vortex core growth. From Refs. 5 and 8, vortex core sizes before breakdown of the order of 0.06 of the blade chord may be expected. Figure 11 indicates that reasonable agreement with the test data resulted only after the core size was increased by a factor of 10, to 0.6 of the blade chord.

However the results of Ref. 5 indicate core growths at encounter of the order of only three times the initial value, which suggests that some other phenomenon in addition to the vortex breakdown may be occurring, for example, the flow separation at close encounter discussed above (Fig. 8). Also a more complete lifting surface representation for the blade in place of a lifting line with local lifting surface correction may be required. Finally the assumptions implicit in the velocity averaging procedure discussed above for determining the wake displacements require further investigation, particularly in the near wake where the velocity distribution depends in part on the time required for the roll-up process to occur. A preliminary analytical investigation of this phenomenon, using the present rotor wake model, is contained in Ref. 13 based on the technique discussed in Ref. 14, in which the wake velocities are determined from the Euler rather than the Biot-Savart relationships. Comprehensive test results on rotors with more than two blades similar to the results of Ref. 11, in which blade loads and wake structures were measured, would provide invaluable guidance in such further analytical effort.

For the four-bladed rotor of Fig. 11 the rapid spanwise variation in circulation due to the close blade vortex first encounter indicated the need for a roll-up schedule which included two center vortices. Figure 12 shows results for the two-bladed rotor using the same roll-up schedule. The results are similar to those of Fig. 2 indicating that, for the two-bladed rotor, the circulation distribution is not particularly sensitive to the assumed inner wake geometry. The effects of vortex core size are negligible for this case. However, representation of the wake by discrete singular vortex filaments occasionally results in convergence problems which are alleviated by introducing a nonsingular vortex core.

Conclusion

The wake geometry and corresponding blade airloads in hovering flight computed using a fast free wake analytical technique are in reasonable agreement with test data. The use of this technique should facilitate expanding the analyses to include real fluids effects on both the vortex structure and blade loads and hopefully contribute to a better understanding of the complex physics of the problem.

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